Soft-Core Model in Nuclear Matter Calculations

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Using a Thomas-Fermi method developed by KUMAR, LE COUTEUR and ROY [1], it is shown here that the two-body soft-core potential suggested by KÖHLER and WAGMARE [2] does not give rise to correct binding energy and equilibrium density in nuclear matter calculations.

I. Introduction

The use of Thomas-Fermi method for calculating nuclear properties has received an impetus by the recent works of BETHE [3] and his coworkers. It is well known that by its very nature the Thomas-Fermi method gives only the overal properties, such as the semi-empirical mass formula, yet the simplicity in a Thomas-Fermi calculation makes it worthwhile, particularly since it can be used as the starting ground for more elaborate Hartree-Fock calculations.

Some time ago, KUMAR, LE COUTEUR and ROY [1] had obtained a Thomas-Fermi method from the K-matrix theory of Brueckner where they had given a simple method for testing the merits of a two-body nuclear potential. It seems natural to us that the first thing one can do with a new two-body potential is to apply the above test to it. If it is found satisfactory, then one can go in for further calculations for nuclear matter and for finite nuclei using either the Thomas-Fermi or the Hartree-Fock method.

As our first choice we have taken the soft-core potential of KÖHLER and WAGMARE [2]. We may also mention that at present we are working on the REID [4] potentials.

II. Calculations and Results

KUMAR, LE COUTEUR and ROY [1] derived an expression for the energy density for a nucleus which, with equal number of neutrons and protons and omitting the Coulomb potential energy, takes the form

$$\mathcal{E} = c\,\varrho^{5/3} - a_1\,\varrho^2 + \frac{2^{4/3}}{3}\,\tau_0\,a_2\,\varrho^{8/3} + a_3(\nabla\varrho)^2 \qquad (1)$$

where $c = 3.6\,\hbar^2/2\,M$ and $\tau_0 = \frac{3}{5}(3/8\,\pi)^{2/3}(2\,\pi)^2$, ($M$ = nucleon mass), and a_1 and a_2 are the first and second moments of Brueckner K-matrix with a_3 given through a_2 in a rather complicated manner. They showed that knowing a_1 and a_2 one could find ϱ_0, the equilibrium density, and λ, the binding energy per particle for nuclear matter, by using the HUGENHOLTZ and VAN HOVE [5] condition that the binding energy per particle in nuclear matter be minimum, i. e.,

$$\frac{\mathrm{d}}{\mathrm{d}\varrho}\left(\frac{\varepsilon_{nm}}{\varrho}\right) = 0\,, \qquad (2)$$

where ε_{nm} is the energy density for nuclear matter. It is easily seen that one can test the merits of a nuclear two-body potential by comparing the values found from the above type of calculations with the presently accepted values [6], namely,

$$\varrho_0 = 0.17\ \mathrm{fm}^{-3} \quad \text{and} \quad \lambda = 16\ \mathrm{MeV}.$$

This is what we propose to do with KÖHLER and WAGMARE's [2] form of a two-body potential

$$v(r) = v_0\,\frac{r^n - c^n}{r^n}\exp(-r^2/r_0^2)\,, \qquad (3)$$

where v_0 represents the strength of the potential, c is the core radius and n determines the "hardness" of the core.

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The parameters a_1 and a_2 in (1) are given as [1]

$$a_1 = -\frac{3}{16}\int K(\boldsymbol{r},\boldsymbol{r}')\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}', \tag{4}$$

$$a_2 = -\frac{3\,q}{128}\int K(\boldsymbol{r},\boldsymbol{r}')\,r^2\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}' \tag{5}$$

with $q = 0.6$ and $K(\boldsymbol{r},\boldsymbol{r}')$ is the Brueckner reaction matrix.

We use the separation method of MOSKOWSKI and SCOTT [7] in which the interaction v is divided into a short-range and a long-range part $v = v_s + v_l$ at a separation distance d. With suitable choice of d we can write the reaction matrix as follows

$$K = v_l + (\Omega_s - D)\,e(Q-1)\,(\Omega_s - 1) + (\Omega_s - 1)\,(e_0 - e)\,(\Omega_s - 1) + \text{higher order terms} \tag{6}$$

where Ω_s is the wave operator and by our choice of d the short range reaction matrix arising from v_s is made equal to zero.

Clearly v_l is the main contribution. The second and third terms are called the Pauli and dispersion terms respectively. Though the contribution due to the dispersion term may not be very small, it will be seen later that it does not affect our results very much. The Pauli term, as we know, will give a very small contribution.

From a look at Eqs. (4), (5) and (6) we see that to find a_1 and a_2 we have to transform K in Eqs. (4) and (5), which are written for coordinate space, to momentum space.

We take a Fourier transform of $K(\boldsymbol{r},\boldsymbol{r}')$ to momentum space

$$K(\boldsymbol{p},\boldsymbol{p}') = \int_{-\infty}^{+\infty} K(\boldsymbol{r},\boldsymbol{r}')\exp\{i(\boldsymbol{p}\cdot\boldsymbol{r}+\boldsymbol{p}'\cdot\boldsymbol{r}')\}\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'. \tag{7}$$

Hence,

$$K(0,0) = \int_{-\infty}^{+\infty} K(\boldsymbol{r},\boldsymbol{r}')\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'. \tag{8}$$

Comparing (8) with (4), we get

$$a_1 = -\frac{3}{16}K(0,0). \tag{9}$$

Also, differentiating Eq. (7) with respect to $\boldsymbol{p}$ twice we have

$$\nabla_p^2 K(\boldsymbol{p},\boldsymbol{p}') = -\int_{-\infty}^{+\infty} K(\boldsymbol{r},\boldsymbol{r}')\,r^2\exp\{i(\boldsymbol{p}\cdot\boldsymbol{r}+\boldsymbol{p}'\cdot\boldsymbol{r}')\}\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}' \tag{10}$$

or

$$\left|\nabla K(\boldsymbol{p},\boldsymbol{p}')\right|_{\substack{p=0\\p'=0}} = -\int_{-\infty}^{+\infty} K(\boldsymbol{r},\boldsymbol{r}')\,r^2\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'. \tag{11}$$

From (11) and (5)

$$a_2 = \frac{3\,q}{128}\left|\nabla_p^2 K(\boldsymbol{p},\boldsymbol{p}')\right|_{\substack{p=0\\p'=0}}. \tag{12}$$

According to MOSZKOWSKI and SCOTT [7] the long-range part of the two-body potential is given as

$$v(\boldsymbol{p},\boldsymbol{p}') = \sum_l (2\,l+1)\,v_l(\boldsymbol{p},\boldsymbol{p}') \tag{13}$$

with

$$v_l(\boldsymbol{p},\boldsymbol{p}') = 4\,\pi\int_d^{\infty} j_l(p\,r)\,v(r)\,j_l(p'\,r)\,r^2\,\mathrm{d}r$$

where $j_l(p\,r)$ denotes the spherical Bessel function.

We know that $j_l(0)$ is zero except for $l=0$. So we need consider the case $l=0$ only.

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We first consider the long-range term v_l of the reaction matrix. From. Eqs. (9) and (14), we have

$$(a_1)_L = -\frac{3}{16}v_0(0,0) = -\frac{3\,\pi}{4}\int_d^{\infty} v(r)\,r^2\,\mathrm{d}r, \tag{15}$$

where $(a_1)_L$ denotes the contribution to a_1 from the long-range term alone. $(a_2)_L$, the contribution to a_2 from the long-range term, is easily found to be

$$(a_2)_L = -\frac{3\,\pi\,q}{32}\int_d^{\infty} v(r)\,r^2\,\mathrm{d}r = \frac{q}{8}(a_1)_L \tag{16}$$

using Eq. (12).

For simplicity of calculation we first consider $(a_1)_L$ and $(a_2)_L$, the main contributory terms to a_1 and a_2. From the Hugenholtz and Van Hove condition (2), we have

$$\frac{d\varepsilon_{nm}}{d\varrho} = \frac{\varepsilon_{nm}}{\varrho} = \lambda \tag{17}$$

where λ is the binding energy per particle of the nuclear matter. By drawing graphs of ε_{nm}/ϱ and $d\varepsilon_{nm}/\varrho$ against ϱ, for integral values of n from 1 to 8 in the two-body potential (3), we find that these graphs intersect for values of ϱ_0 ranging between 1.55 and 1.65 and of λ between 70 and 100 which are much too high to be reasonable values. This probably means that the two-body potential of Köhler and Wagmare is too soft.

If we now consider the contributions to a_1 and a_2 due to the Pauli and dispersion terms in the reaction matrix, we see that these contributions do not improve the situation. The contrtibutions due to the Pauli terms are very small, and the contributions due to the dispersion term do not improve the situation very much. In fact, it is found that to get the correct values of $\varrho_0 = 0.17$ and $\lambda = 16$ MeV we have to increase the value of a_2 by $3\,a_2$. Hence it is very unlikely that one will get correct results by adding contributions from the Pauli and dispersion terms to the main term $(a_2)_L$, the long-range part of the interaction. Since the soft-core two-body potential used here does not give reasonable values for the equilibrium density (ϱ_0) and the binding energy per particle in nuclear matter (λ), we think it unlikely to be of much use for further calculations for nuclear matter or finite nuclei.

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Messungen des Anlagerungskoeffizienten von Elektronen in Sauerstoff *

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The attachment coefficient η for the formation of negative ions in low energy electron swarms was measured over an E/p-range of 0.1 ... 30 V/Torr cm by a new and accurate method. Thus earlier measurements up to 54 Torr of other authors could be extended to 880 Torr.

The shape of the minimum in the η/p-curves between the three-body process ($e+2\,O_2 \rightarrow O_2^- + O_2$) and the dissociative process ($e+O_2 \rightarrow O^- + O$) and its shift to higher E/p with increasing pressure was measured. Behind the minimum a maximum at $E/p=14$ was found. Between the minimum and this maximum the dissociative process is predominant but the three-body process is still of influence. For $E/p > 14$ the η/p-values are slowly decreasing with increasing E/p.

For the higher pressures above 44 Torr deviations from the relation η proportional to p^2 were found for the three-body process. These deviations are discussed.

Es wird die Bildung stabiler negativer Sauerstoffionen bei der Drift von Elektronen im homogenen elektrischen Feld untersucht. Durch die Verwendung einer neuen, genaueren Meßmethode werden die früheren Ergebnisse anderer Autoren [1–9] erweitert.

Der Anlagerungskoeffizient η ist definiert als die Anzahl von Anlagerungen, die ein Elektron eines Elektronenschwarms im Mittel pro cm Driftweg erleidet. $\eta\,dx$ ist die Wahrscheinlichkeit der Anlagerung auf der Strecke dx.

Nach CHANIN, PHELPS und BIONDI [1] treten beim Stoß von Elektronen mit Sauerstoffmolekülen zwei unterschiedliche Anlagerungsprozesse auf, in denen die beiden negativen Ionen O^- oder O_2^- gebildet

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